Rotation and shift of a camera with thin lens -- in reaction to focal plane shutter operation

Out[2]=

Center of gravity:

In[3]:=

```
CoG = m0 r0 + m1 r1 - m2 r2 == 0
```

Out[3]=

$m0\,r0 + m1\,r1 - m2\,r2 == 0$

Out[4]=

m0 shutter mass, m1 body mass, m2 lens mass idealized point masses. r0, r1 and r2 the distance to the center of gravity of respective masses

Out[5]=

Moments of inertia:

In[6]:=

```
J0 = m0 r0^2
```

Out[6]=

$m0\,r0^2$

In[7]:=

```
J1 = m1 r1^2
```

Out[7]=

$m1\,r1^2$

In[8]:=

```
J2 = m2 r2^2
```

Out[8]=

$m2\,r2^2$

In[9]:=

Out[9]=

```
Preservation of rotational
  momentum when shutter operates:
```

In[10]:=

```
PRM = -J0 p0 + J1 p1 + J2 p1 == 0
```

Out[10]=

$$-m0\,p0\,r0^{2} + m1\,p1\,r1^{2} + m2\,p1\,r2^{2} == 0$$

Out[11]=

```
where p0 is the angle of rotation of the shutter (
  around CoG) and p1 is the angle of rotation
  of body and lens (which rotate in common)
```

In[12]:=

Out[12]=

```
Preservation of center of gravity (of
  linear momentum) in z direction (height):
```

In[19]:=

```
h = s (m0 + m1 + m2) - m0 r0 p0 + m1 r1 p1 - m2 r2 p1 == 0
```

Out[19]=

```
-m0 p0 r0 + m1 p1 r1 - m2 p1 r2 + (m0 + m1 + m2) s == 0
```

Out[14]=

```
where s is the shift by which the CoG moves
  off the connection line between m1 and m2,
  assuming the shutter mass m0 initially was on
  that line and after operation, it rotated off
  that line by an angle p0+p1. s is the amount
  by which the outer body+lens moves in height
```

In[15]:=

Out[15]=

```
formula h applies to small rotation angles p only.
  Note that while p1 is small, p0 can exceed 10
  °. However, this induces a small error still.
```

In[43]:=

Out[43]=

Approximate shutter move
in z direction y (note: p1<<p0):

In[32]:=

```
Smove0 = y == r0 (p0 + p1)
```

Out[32]=

$y == (p0 + p1)\, r0$

In[40]:=

```
Smove1 = y == r0 p0
```

Out[40]=

$y == p0\, r0$

In[41]:=

Out[41]=

And now solve for the shift s of body:

In[44]:=

```
Simplify[Solve[h /. Solve[CoG, r2], s] /.
   Solve[Smove0, p0]] /. {m0 + m1 + m2 → M}
```

Out[44]=

$$\left\{\left\{\left\{s \to \frac{m0\, y}{M}\right\}\right\}\right\}$$

In[45]:=

Out[45]=

Blur b due to rotation with focal length f:

In[46]:=

```
b = f p1
```

Out[46]=

$f\, p1$

In[47]:=

Out[47]=

```
thin lens focal length is sum
  of distances of shutter and lens to CoG:
```

In[48]:=

```
f = r0 + r2
```

Out[48]=

$$r0 + r2$$

In[58]:=

```
brot =
 Simplify[b /. Solve[PRM, p1] /. Solve[Smove1, p0] /.
   Solve[CoG /. m0 → 0, r2]]
```

Out[58]=

$$\left\{\left\{\left\{\frac{m0\ r0\ (m2\ r0 + m1\ r1)\ y}{m1\ (m1 + m2)\ r1^2}\right\}\right\}\right\}$$

In[61]:=

Out[61]=

```
Now, if the shutter is in
  the center of gravity of the body:
```

In[62]:=

```
Simplify[brot /. r0 → r1]
```

Out[62]=

$$\left\{\left\{\left\{\frac{m0\ y}{m1}\right\}\right\}\right\}$$

In[63]:=

Out[63]=

```
Conclusion: rotational blur due to shutter shift
  is always larger than the body translational
  shift s and about independent on the weight or
  focal length of the lens used. Surprising, eh?
```
